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# 3-TOTAL EDGE SUM CORDIAL LABELING BY APPLYING UNION OPERATION ON SOME GRAPHS.

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**Abstract** In this paper, we have discussed a variant of edge sum cordial labeling of graphs known as 3-Total edge sum cordial labeling of Graphs. Unlike in 3-Total sum cordial labeling the roles of vertices and edges are interchanged. In this paper, this labeling is investigated by applying union operation on some of the graphs. We investigate union of two path graphs on this concept.

**MSC:** 05C76, 05C78.
**Keywords:** Cordial labeling, Edge sum cordial labeling, 3-Total edge sum cordial labeling, 3-Total edge sum cordial graphs.

## Introduction

The graphs considered here are finite, undirected and simple. For all other terminology and notation follow Harrary [3]. Let $G\ (V, E)$ be a graph where the symbols $V(G)$ and $E(G)$ denotes the vertex set and edge set respectively. Graphs $G_1$ and $G_2$ have disjoint vertex set $V_1$ and $V_2$ and edge set $E_1$ and $E_2$ respectively. The union of $G_1$ and $G_2$ is the graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.If the vertices or edges or both of the graph are assigned values subject to certain condition it is known as graph labeling. Graph labeling serves as a frontier between number theory and application of graphs. A dynamic survey of graph labeling is regularly updated by Gallian [2]. Cordial graphs was introduced by Cahit [1] as a weaker version of both graceful and harmonious graphs. The concept of sum cordial labeling of graph was introduced by Shiama J. [4]. The concept of 3-Total super sum cordial labeling of graphs was introduced by Tenguria Abha and Verma Rinku [5]. The concept of 3-Total super sum cordial labeling for union of graphs was introduced by Tenguria Abha and Verma Rinku [6]. The concept of 3-Total super product cordial labeling of graphs was introduced by Tenguria Abha and Verma Rinku [7]. The concept of 3-Total edge sum cordial labeling of graphs was introduced by Tenguria Abha and Verma Rinku [8].The edge product cordial labeling of graphs was introduced by Vaidya S. K. and Barasara C.M. [9]. We will give brief summary of definition which are useful for the present investigation.

**Definition 1.1:** Let $G$ be a graph. Let $f:V(G)\rightarrow\{0,1,2\}$ For each edge $uv$ assign the label $[f(u)+f(v)](\mathrm{mod}\,3)$. Then the map $f$ is called 3-Total sum cordial labeling of $G$, if $|f(i)-f(j)|\leq 1: i,j\in\{0,1,2\}$. where $f(x)$ denotes the total number of vertices and edges labeled with $x=\{0,1,2\}$.

**Definition 1.2:** For graph $G$ the edge labeling function is defined as $f:E(G)\rightarrow\{0,1,2\}$ and induced vertex labeling $f^*:V(G)\rightarrow\{0,1,2\}$ is given as if $e_1,e_2,...,e_n$ are the edge incident on vertex $v$ then $f^*(v)=f(e_1)+_3 f(e_2)+_3 ...+_3 f(e_n)$ Then the map $f$ is called 3-Total edge sum cordial labeling of a graph $G$ if $|f(i)-f(j)|\leq 1: i,j\in\{0,1,2\}$. where $f(x)$ denotes the total number of vertices and edges labeled with $x=\{0,1,2\}$.

**Definition 1.3:** Let $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ be two graphs then their union denoted by $G_1\cup G_2=(V_1\cup V_2, E_1\cup E_2)$.

## 2. Main Result

**Theorem 2.1:** The graph $P_m\cup P_n$ is 3-Total edge sum cordial.
Proof. Let $e_1,e_2,...,e_{m-1}$ be edges of path $P_m$ and $e'_1,e'_2,...,e'_{n-1}$ be edges of path $P_n$.

**Case I:** $m\equiv 0(\mathrm{mod}\,3)$ and $n\equiv 0(\mathrm{mod}\,3)$.
Let $m=3p$ and $n=3t$.
Define

$$f(e_{i+1})=0; 0\leq i<p,$$
$$f(e_{i+p+1})=2; 0\leq i<2p-1,$$
$$f(e'_{i+1})=0; 0\leq i<t,$$
$$f(e'_{i+t+1})=1; 0\leq i<2t-1,$$

Hence $f$ is 3-Total edge sum cordial.

**Case II:** $m\equiv 0(\mathrm{mod}\,3)$ and $n\equiv 1(\mathrm{mod}\,3)$.
Let $m=3p$ and $n=3t+1$.
Define

$$f(e_{2i+1})=1; 0\leq i<p,$$
$$f(e_{2i+2})=2; 0\leq i<p,$$

$$f(e_{2p+1+i}) = 2; 0 \le i < p-1,$$

Assign

$$f(e'_{n-1}) = 0,$$
$$f(e'_{n-2}) = 1,$$
$$f(e'_{n-3}) = 2,$$

Define

$$f(e'_{3i+1}) = 2; 0 \le i < t-1,$$
$$f(e'_{3i+2}) = 2; 0 \le i < t-1,$$
$$f(e'_{3i+3}) = 1; 0 \le i < t-1.$$

Hence $f$ is 3-Total edge sum cordial.

**Case III:** $m \equiv 0(\bmod 3)$ and $n \equiv 2(\bmod 3)$.
Let $m = 3p$ and $n = 3t+2$.
where $t \ne 0.$,

Define

$$f(e_{2i+1}) = 1; 0 \le i < p,$$
$$f(e_{2i+2}) = 2; 0 \le i < p,$$
$$f(e_{2p+1+i}) = 2; 0 \le i < p-1,$$

Assign

$$f(e'_{n-1}) = 1,$$
$$f(e'_{n-2}) = 2,$$
$$f(e'_{n-3}) = 1,$$
$$f(e'_{n-4}) = 2,$$

Define

$$f(e'_{3i+1}) = 2; 0 \le i < t-1,$$
$$f(e'_{3i+2}) = 2; 0 \le i < t-1,$$
$$f(e'_{3i+3}) = 1; 0 \le i < t-1.$$

Hence $f$ is 3-Total edge sum cordial.

**Case IV:** $m \equiv 1(\bmod 3)$ and $n \equiv 0(\bmod 3)$.
Let $m = 3p+1$ and $n = 3t$.
Assign

$$f(e_{m-1}) = 0,$$
$$f(e_{m-2}) = 1,$$
$$f(e_{m-3}) = 2,$$

Define

$$f(e_{3i+1}) = 2; 0 \le i < p-1,$$
$$f(e_{3i+2}) = 2; 0 \le i < p-1,$$
$$f(e_{3i+3}) = 1; 0 \le i < p-1,$$
$$f(e'_{2i+1}) = 1; 0 \le i < t,$$
$$f(e'_{2i+2}) = 2; 0 \le i < t,$$
$$f(e'_{2t+1+i}) = 2; 0 \le i < t-1.$$

Hence $f$ is 3-Total edge sum cordial.

**Case V:** $m \equiv 1 (\bmod 3)$ and $n \equiv 1 (\bmod 3)$.
Let $m = 3p+1$ and $n = 3t+1$.
Define

$$f(e_{3i+1}) = 2; 0 \le i < p,$$
$$f(e_{3i+2}) = 1; 0 \le i < p,$$
$$f(e_{3i+3}) = 1; 0 \le i < p,$$

Assign

$$f(e'_{n-1}) = 0,$$
$$f(e'_{n-2}) = 1,$$
$$f(e'_{n-3}) = 2,$$

Define

$$f(e'_{3i+1}) = 2; 0 \le i < t-1,$$
$$f(e'_{3i+2}) = 2; 0 \le i < t-1,$$
$$f(e'_{3i+3}) = 1; 0 \le i < t-1.$$

Hence $f$ is 3-Total edge sum cordial.

**Case VI:** $m \equiv 1 (\bmod 3)$ and $n \equiv 2 (\bmod 3)$.
Let $m = 3p+1$ and $n = 3t+2$.
where $t \neq 0$. ,
Assign

$$f(e_{m-1}) = 0,$$
$$f(e_{m-2}) = 1,$$
$$f(e_{m-3}) = 2,$$

Define

$$f(e_{3i+1}) = 2; 0 \le i < p-1,$$
$$f(e_{3i+2}) = 2; 0 \le i < p-1,$$

$$f(e_{3i+3}) = 1; 0 \le i < p-1,$$

Assign

$$f(e'_{n-1}) = 1,$$
$$f(e'_{n-2}) = 2,$$
$$f(e'_{n-3}) = 1,$$
$$f(e'_{n-4}) = 2,$$

Define

$$f(e'_{3i+1}) = 2; 0 \le i < t-1,$$
$$f(e'_{3i+2}) = 2; 0 \le i < t-1,$$
$$f(e'_{3i+3}) = 1; 0 \le i < t-1.$$

Hence $f$ is 3-Total edge sum cordial.

**Case VII:** $m \equiv 2(\bmod 3)$ and $n \equiv 0(\bmod 3)$.
Let $m = 3p+2$ and $n = 3t$.
where $p \ne 0$ ,
Assign

$$f(e_{m-1}) = 1,$$
$$f(e_{m-2}) = 2,$$
$$f(e_{m-3}) = 1,$$
$$f(e_{m-4}) = 2,$$

Define

$$f(e_{3i+1}) = 2; 0 \le i < p-1,$$
$$f(e_{3i+2}) = 2; 0 \le i < p-1,$$
$$f(e_{3i+3}) = 1; 0 \le i < p-1,$$
$$f(e'_{2i+1}) = 1; 0 \le i < t,$$
$$f(e'_{2i+2}) = 2; 0 \le i < t,$$
$$f(e'_{2t+1+i}) = 2; 0 \le i < t-1.$$

Hence $f$ is 3-Total edge sum cordial.

**Case VIII:** $m \equiv 2(\bmod 3)$ and $n \equiv 1(\bmod 3)$.
Let $m = 3p+2$ and $n = 3t+1$.
where $p \ne 0.$ ,
Assign

$$f(e_{m-1}) = 1,$$
$$f(e_{m-2}) = 2,$$

$$f(e_{m-3}) = 1,$$
$$f(e_{m-4}) = 2,$$

Define

$$f(e_{3i+1}) = 2; 0 \le i < p-1,$$
$$f(e_{3i+2}) = 2; 0 \le i < p-1,$$
$$f(e_{3i+3}) = 1; 0 \le i < p-1,$$

Assign

$$f(e'_{n-1}) = 0,$$
$$f(e'_{n-2}) = 1,$$
$$f(e'_{n-3}) = 2,$$

Define

$$f(e'_{3i+1}) = 2; 0 \le i < t-1,$$
$$f(e'_{3i+2}) = 2; 0 \le i < t-1,$$
$$f(e'_{3i+3}) = 1; 0 \le i < t-1.$$

Hence $f$ is 3-Total edge sum cordial.

**Case IX:** $m \equiv 2(\bmod 3)$ and $n \equiv 2(\bmod 3)$.
Let $m = 3p+2$ and $n = 3t+2$.
where $p, t \ne 0.$ ,
Assign

$$f(e_{m-1}) = 1,$$
$$f(e_{m-2}) = 2,$$
$$f(e_{m-3}) = 1,$$
$$f(e_{m-4}) = 2,$$

Define

$$f(e_{3i+1}) = 2; 0 \le i < p-1,$$
$$f(e_{3i+2}) = 2; 0 \le i < p-1,$$
$$f(e_{3i+3}) = 1; 0 \le i < p-1,$$

Assign

$$f(e'_{n-1}) = 1,$$
$$f(e'_{n-2}) = 2,$$
$$f(e'_{n-3}) = 1,$$
$$f(e'_{n-4}) = 2,$$

Define

$$f(e'_{3i+1}) = 2; 0 \le i < t-1,$$

$$f(e'_{3i+2}) = 2; 0 \le i < t-1,$$

$$f(e'_{3i+3}) = 1; 0 \le i < t-1.$$

Hence $f$ is 3-Total edge sum cordial.

**Illustration:**


**Fig 1** : 3- Total edge sum cordial labeling of $P_5 \cup P_7$.

**Illustration:**

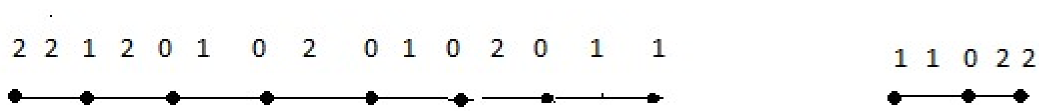


**Fig 2 :** 3- Total edge sum cordial labeling of $P_8 \cup P_3$.

**Table 1:** Vertex and edge conditions for 3-Total edge sum cordial labeling of $P_m \cup P_{n.}$

| Case | Vertex Condition | Edge Condition | $f(i) = v_f(i) + e_f(i)$ |
|---|---|---|---|
| m=3p & n=3t | $v_f(0) = p + t$<br>$v_f(1) = 2p$<br>$v_f(2) = 2t$ | $e_f(0) = p + t$<br>$e_f(1) = 2t - 1$<br>$e_f(2) = 2p - 1$ | $f(0) = 2p + 2t$<br>$f(1) = 2p + 2t$-1<br>$f(2) = 2p + 2t$-1 |
| m=3p & n=3t+1 | $v_f(0) = 2p + 2t - 1$<br>$v_f(1) = p + t$<br>$v_f(2) = 2$ | $e_f(0) = 1$<br>$e_f(1) = p + t$<br>$e_f(2) = 2p + 2t - 2$ | $f(0) = 2p + 2t$<br>$f(1) = 2p + 2t$<br>$f(2) = 2p + 2t$ |
| m=3p & n=3t+2 | $v_f(0) = 2p + 2t$<br>$v_f(1) = p + t$<br>$v_f(2) = 2$ | $e_f(0)$ =0<br>$e_f(1) = p + t + 1$<br>$e_f(2) = 2p + 2t - 1$ | $f(0) = 2p + 2t$<br>$f(1) = 2p + 2t$+1<br>$f(2) = 2p + 2t + 1$ |
| m=3p+1 & n=3t | $v_f(0) = 2p + 2t - 1$<br>$v_f(1) = p + t$<br>$v_f(2) = 2$ | $e_f(0) = 1$<br>$e_f(1) = p + t$<br>$e_f(2) = 2p + 2t - 2$ | $f(0) = 2p + 2t$<br>$f(1) = 2p + 2t$<br>$f(2) = 2p + 2t$ |
| m=3p+1 & n=3t+1 | $v_f(0) = 2p + 2t - 1$<br>$v_f(1) = t + 1$<br>$v_f(2) = p + 2$ | $e_f(0) = 1$<br>$e_f(1) = 2p + t$<br>$e_f(2) = p + 2t - 1$ | $f(0) = 2p + 2t$<br>$f(1) = 2p + 2t$+1<br>$f(2) = 2p + 2t + 1$ |
| m=3p+1 & n=3t+2 | $v_f(0) = 2p + 2t + 1$<br>$v_f(1) = p + t$ | $e_f(0)$ =1<br>$e_f(1) = p + t + 1$ | $f(0) = 2p + 2t + 2$<br>$f(1) = 2p + 2t + 1$ |

| | | | |
|---|---|---|---|
| | $v_f(2) = 2$ | $e_f(2) = 2p + 2t - 1$ | $f(2) = 2p + 2t + 1$ |
| m=3p+2 & n=3t | $v_f(0) = 2p + 2t$<br>$v_f(1) = p + t$<br>$v_f(2) = 2$ | $e_f(0) = 0$<br>$e_f(1) = p + t + 1$<br>$e_f(2) = 2p + 2t - 1$ | $f(0) = 2p + 2t$<br>$f(1) = 2p + 2t + 1$<br>$f(2) = 2p + 2t + 1$ |
| m=3p+2 & n=3t+1 | $v_f(0) = 2p + 2t + 1$<br>$v_f(1) = p + t$<br>$v_f(2) = 2$ | $e_f(0) = 1$<br>$e_f(1) = p + t + 1$<br>$e_f(2) = 2p + 2t - 1$ | $f(0) = 2p + 2t + 2$<br>$f(1) = 2p + 2t + 1$<br>$f(2) = 2p + 2t + 1$ |
| m=3p+2 & n=3t+2 | $v_f(0) = 2p + 2t + 2$<br>$v_f(1) = p + t$<br>$v_f(2) = 2$ | $e_f(0) = 0$<br>$e_f(1) = p + t + 2$<br>$e_f(2) = 2p + 2t$ | $f(0) = 2p + 2t + 2$<br>$f(1) = 2p + 2t + 2$<br>$f(2) = 2p + 2t + 2$ |

## 3. Conclusion

We have investigated 3-Total edge sum cordial labeling by applying union operation on Path graphs. To investigate analogous results for different graphs or by applying different operations on graphs as well as in the context of various graph labeling problems is an open area of research.

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