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An Integral of the Multivariable H-Function & M-Series with application

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Abstract

In present investigation to obtain an integral for the product of Fox's H-function. The generalized polynomials given by Srivastava[22]. The H-function of 'r' variable introduced by Srivastava and Panda [21]. We gave representation of the generalized M-series in term of Mittag-Leffter function and generalized hypergeometric function introduced by Manoj Sharma and jain [13]. M-series and both form of the representations with these various functions and polynomials, On the other hand, the multivariable H-function occurring in our main results involving different special functions can be obtained by merely by specializing the parameters in the presented formulas can be reduced under various particular cases.

Keywords and phrases: Fox's H-function, general class of multivariable polynomials, Multivariable H-function, M-series, Hypergeometric function.

I. INTRODUCTION

The M-series is particular case of the H-function of Innayat-Hussain [5]. It is special role in the application of fractional calculus operators and in the solution of multivariable H-function and polynomials Sharma and Jain introduced [13] the general M-series as the function defined by means of the power series.

$$ {}_{P}^{\alpha}M_{q}^{\beta}\left(a_1,\dots,a_p;b_1,\dots,b_q:z\right) = \sum_{s'=0}^{\infty}\frac{(a_1)_{s'},\dots,(a_p)_{s'}}{(b_1)_{s'},\dots,(b_q)_{s'}}\,\frac{z^{s'}}{\Gamma(\beta s'+\alpha)}, \quad (1.1)$$

where, $$F(s') = \frac{\prod_{j=1}^{p}(a_j)_{s'}\,z^{s'}}{\prod_{j=1}^{q}(b_j)_{s'}\Gamma(\beta s'+\alpha)} \quad (1.1a)$$

where $\alpha,\beta \in \mathbb{C}$ and $\Re e(\alpha)$b $\geq 0, \Re e\,(\beta) \geq 0$ and $\left(a_j\right)_{s'}, \left(b_j\right)_{s'}$ are Pochhammer's symbols. The series [1.1] is defined when none of the denominator parameter $\left(b_j\right)_{s'}$, (j = 1, …,q) is a negative integer or zero.

Here we take α = 0 then the above series converted Sharma's M-series [12]

$$ {}_{P}M_{q}^{\beta}\left(a_1,\dots,a_p;b_1,\dots,b_q:z\right) = \sum_{s'=0}^{\infty}\frac{(a_1)_{s'},\dots,(a_p)_{s'}}{(b_1)_{s'},\dots,(b_q)_{s'}}\,\frac{z^{s'}}{\Gamma(\beta s'+1)}, \quad (1.1b)$$

where, $$L(s') = \frac{\prod_{j=1}^{p}(a_j)_{s'}\,z^{s'}}{\prod_{j=1}^{q}(b_j)_{s'}\Gamma(\beta s'+1)} \quad (1.1c)$$

Condition-1 when $\alpha,\beta = 1$, then we have to so the generalized series ${}_{P}^{1}M_{q}^{1}(z)$ become to the generalized hypergeometric function [5]

$${}_P^1M_q^1\left(a_1,\ldots,a_p;b_1,\ldots,b_q:z\right) = \sum_{s'=0}^{\infty}\frac{(a_1)_{s'},\ldots,(a_p)_{s'}}{(b_1)_{s'},\ldots,(b_q)_{s'}}\frac{z^{s'}}{\Gamma(s'+1)}\ {}_PF_q[z] \qquad (1.1d)$$

where, $G(s') = \frac{\prod_{j=1}^{p}(a_j)_{s'}\,z^{s'}}{\prod_{j=1}^{q}(b_j)_{s'}\Gamma(s'+1)}$. (1.1e)

Condition -2 when there is no upper and lower parameters hence p = q = 0 and α = 1 then we have

$${}_0^1M_0^{\beta}\left(\ldots;\ldots:z\right) = \sum_{s'=0}^{\infty}\frac{z^{s'}}{\Gamma(\beta s'+1)} = E_{\beta}[z] \qquad (1.1f)$$

The series ${}_0^1M_0^{\beta}[z]$ becomes Mittage-Leffter function represented by $E_{\beta}[z]$ [8]

where,

$$T(s') = \frac{z^{s'}}{\Gamma(\beta s'+1)} \qquad (1.1g)$$

The Fox's (1961) H-function[4]

$$H_{U,V}^{S,N}\left[L\left|\begin{matrix}[k_P,K_P]\\ [f_Q,F_Q]\end{matrix}\right.\right] = \sum_{G!=0}^{\infty}\sum_{g=1}^{S}\frac{(-1)^G}{G!\,F_g}\xi(\eta_G)L^{\eta_G} \qquad (1.2)$$

where

$$\xi(\eta_G) = \frac{\prod_{j=1,j\neq g}^{S}\Gamma(f_j-F_j\,\eta_G)\prod_{j=1}^{N}\Gamma(1-k_j+K_j\eta_G)}{\prod_{j=S+1}^{Q}\Gamma(1-f_j+F_j\,\eta_G)\prod_{j=N+1}^{P}\Gamma(k_j-K_j\eta_G)},$$

$$\eta_G = \frac{(f_g+G)}{F_g} \qquad (1.2a)$$

A general class of multivariable polynomials defined by H. M. Srivastava and M. Garg (1987) as follows[15]

$$S_n^{m_1,\ldots,m_k}[y_1,\ldots,y_k] = \sum_{r_1,\ldots,r_k=0}^{m_1r_1+\cdots+m_kr_k\leq n}(-n)_{m_1r_1+\cdots+m_kr_k}$$

$$A\,(n;r_1,\ldots,n_k)\frac{y_1^{r_1}}{r_1!},\ldots,\frac{y_k^{r_k}}{r_k!}. \qquad (1.3)$$

where $m_1,\ldots,m_k \in \mathbb{N}_0 = (0,1,2,\ldots)$ and The coefficients $A(n;r_1,\ldots,\ r_k)$ $(r_i \in \mathbb{N}_0; i = 1,\ldots,k)$ arbitrary constants, real or complex.

H.M. Srivastava has defined and introduced the general polynomials [19]

$$S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}[y_1,\ldots,y_k] = \sum_{r_1=0}^{[n_1/m_1]},\ldots,\sum_{r_k=0}^{[n_k/m_k]}(-n_1)_{m_1r_1},\ldots,(-n_k)_{m_kr_k}$$

$$A\,(n_1,r_1;\ldots;n_k,r_k)\frac{y_1^{r_1}}{r_1!},\ldots,\frac{y_k^{r_k}}{r_k!}. \qquad (1.4)$$

where,$n_i = 0,1,2,\ldots$ $\forall$ i ∈ $(1,\ldots,k)$; $m_1,\ldots,m_k$ are arbitrary positive integers and the coefficient $A\,(n_1,r_1;\ldots;n_k,r_k)$ are arbitrary constants, real or complex.

The multivariable H-function introduced by Srivastava and panda [20]

$$H^{0,n:\,m_1,n_1;\ldots;m_s,n_s}_{p,q:\,p_1,q_1;\ldots;p_s,q_s}\left[\begin{matrix} z_1 \\ \vdots \\ z_s \end{matrix}\middle| \begin{matrix} (\mathrm{a_j};\ \alpha_j^1,\ldots,\alpha_j^s)_{1,p} : (c_j^1,\gamma_j^1)_{1,p_1};\ldots;(c_j^s,\gamma_j^s)_{1,p_s} \\ (\mathrm{b_j};\ \beta_j^1,\ldots,\beta_j^s)_{1,q} : (d_j^1,\delta_j^1)_{1,q_1};\ldots;(d_j^s,\delta_j^s)_{1,q_s} \end{matrix}\right]$$

$$= \frac{1}{(2\pi\omega)^s}\int_{\xi_1}\ldots\int_{\xi_s}\Psi(\lambda_1,\ldots,\lambda_s)\,\phi_1(\lambda_1),\ldots,\phi_s(\lambda_s) z_1^{\lambda_1},\ldots,z_s^{\lambda_s}$$

$$d\lambda_1,\ldots,d\lambda_s. \qquad (1.5)$$

where, $\omega = \sqrt{-1}$; and

$$\Psi(\lambda_1,\ldots,\lambda_s) = \frac{\prod_{j=1}^{n}\Gamma\left(1-a_j+\sum_{i=1}^{s}\alpha_j^{(i)}\lambda_i\right)}{\prod_{j=n+1}^{p}\Gamma\left(a_j-\sum_{i=1}^{s}\alpha_j^{(i)}\lambda_i\right)\prod_{j=1}^{q}\Gamma\left(1-b_j+\sum_{i=1}^{s}\beta_j^{(s)}\lambda_i\right)}, \qquad (1.5a)$$

$$\phi_i(\lambda_i) = \frac{\prod_{j=1}^{n_i}\Gamma\left(1-c_j^{(i)}+\gamma_j^{(i)}\lambda_i\right)\prod_{j=1}^{m_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}\lambda_i\right)}{\prod_{j=n_i+1}^{p_i}\Gamma\left(c_j^{(i)}-\gamma_j^{(i)}\lambda_i\right)\prod_{j=m_i+1}^{q_i}\Gamma\left(1-d_j^{(i)}+\delta_j^{(i)}\lambda_i\right)} \qquad (1.5b)$$

$(i = 1,2,\ldots,s)$

where, $n, p, q, m_i, n_i, p_i, q_i,\ i$ are non-negative integer. Such that $0 \le n \le p, q \ge 0, 1 \le m_i \le q_i$ and $0 \le n_i \le p_{i,} \forall\, i \in 1,\ldots,s$ and $\alpha_i, \beta_i, \gamma_i$ and δ_i are all positive and the inequalities in the contour L_j lies in the complex ξ_j − plane is of Mellin-Barnes type which start at the point $\tau_j - \omega^\infty$ and terminate at the point $\tau_j + \omega^\infty$with $\tau_j \in \Re e = (-\infty,+\infty), (j = 1,\ldots,s)$

Such that all the poles of $\Gamma_{i=1}^{s}(d_j^{(i)} - \delta_j^{(i)}\lambda_i); i = 1,2,\ldots,s,\ \Gamma_{i=1}^{s}(1 - a_j + \alpha_j^{(i)}\lambda_i)$ and $(j = 1,\ldots,n)$ are to the left of ξ_i

$$\nabla_i = \sum_{j=1}^{p}\alpha_j + \sum_{j=1}^{p_i}\gamma_j - \sum_{j=1}^{q}\beta_j + \sum_{j=1}^{q_i}\delta_j \le 0 \qquad (1.5c)$$

$$\Re_i = -\sum_{j=n+1}^{p}\alpha_j + \sum_{j=1}^{q}\beta_j + \sum_{j=1}^{n_i}\gamma_j - \sum_{j=n_i+1}^{p_i}\gamma_j + \sum_{j=1}^{m_i}\delta_j - \sum_{j=m_i+1}^{q_i}\delta_j > 0 \qquad (1.5d)$$

Saxena introduced of an integral involving G-function (9) following

$$\int_0^\infty k^{1-\mu}\ (R+Sk+Tk^2)^{\mu-\frac{3}{2}}\,dk = \sqrt{\frac{\pi}{T}}\left(S+2\sqrt{TR}\right)^{\mu-1}\frac{\Gamma(1-u)}{\Gamma(\frac{3}{2}-u)} \qquad (1.6)$$

II MAIN INTEGRAL

$$\int_0^\infty k^{1-\mu}\ (R+Sk+Tk^2)^{\ \mu-3/2}\,H^{S,N}_{U,V}\left[\left(\frac{k}{(R+Sk+Tk^2)}\right)^p\middle|\begin{matrix}[k_P,K_P]\\ [f_Q,F_Q]\end{matrix}\right]$$

$$S^{M_1,\ldots,M_\ell}_{N_1,\ldots,N_\ell}\left[w_1\left(\frac{k}{(R+Sk+Tk^2)}\right)^{s_1},\ldots,w_\ell\left(\frac{k}{(R+Sk+Tk^2)}\right)^{s_\ell}\right]\ {}^{\alpha}_{p}M^{\beta}_{q}\ \left[\left(\frac{k}{(R+Sk+Tk^2)}\right)^{\gamma}\right]$$

$$S^{\alpha_1,\ldots,\alpha_\ell}_{V}\left[w_1\left(\frac{k}{(R+Sk+Tk^2)}\right)^{s_1},\ldots,w_\ell\left(\frac{k}{(R+Sk+Tk^2)}\right)^{s_\ell}\right]$$

$$H\left[y_1\left(\frac{k}{(R+Sk+Tk^2)}\right)^{\rho_1},\ldots,y_1\left(\frac{k}{(R+Sk+Tk^2)}\right)^{\rho_1}\middle|\begin{matrix}(a_j;\ \alpha_j^{(1)},\ldots,\alpha_j^{(s)})_{1,p} :\ldots;\ldots;\ldots \\ (b_j;\ \beta_j^{(1)},\ldots,\beta_j^{(1)})_{1,q} :\ldots;\ldots;\ldots\end{matrix}\right] dz$$

$$= \sqrt{\frac{\pi}{T}} \sum_{G!=0}^{\infty} \sum_{g=1}^{S} \sum_{S'=0}^{\infty} \sum_{L_1=0}^{[N_1/M_1]}, \dots, \sum_{L_\ell=0}^{[N_\ell/M_\ell]} \sum_{k_1+\dots+k_\ell}^{\alpha_1 k_1+\dots+\alpha_\ell k_\ell \le V} \frac{(-1)^G}{G!\,F_g} F(s')$$

$$\xi(\eta_G)(-V)_{\alpha_1 k_1+\dots+\alpha_\ell k_\ell} A(V;k_1,\dots,k_\ell) B[N_1,L_1;\dots;N_\ell,L_\ell] \frac{(-N_1)_{M_1,L_1}}{L_1},\dots,\frac{(-N_\ell)_{M_\ell,L_\ell}}{L_\ell} \frac{W_1^{L_1+,K_1}}{L_1},\dots,\frac{W_\ell^{L_\ell+,K_\ell}}{L_\ell} (S+$$

$$2\sqrt{TR})^{\mu-\varphi\eta_G-\gamma s'-\sum_{i=1}^{\ell} s_i(L_i+,K_i)-1} H_{B+1,D+1:\,p_1,q_1;\dots;p_s,q_s}^{0,\Omega+1:\,m_1,n_1;\dots;m_s,n_s}$$

$$\left[\left|\begin{matrix} (-\mu-\varphi\eta_G-\gamma s'-\sum_{i=1}^{\ell} s_i(L_i+,K_i);\rho_1,\dots,\rho_r),(a_j;\alpha_j^{(1)},\alpha_j^{(s)})_{1,p}:\dots;\dots;\dots \\ (b_j;\beta_j^{(1)},\beta_j^{(s)})_{1,q}(-\mu-\varphi\eta_G-\gamma s'-\sum_{i=1}^{\ell} s_i(L_i+K_i)-1/2;\ \rho_1,\dots,\rho_r):\dots;\dots;\dots \end{matrix}\right.\right]$$

$\Re e(R)>0, \Re e(S)>0, (T)>0$ *and* $, L_i>0\ (i=1,\dots,\ell),$ $\psi\left[\min\left\{\Re e\left(\frac{f_j}{F_j}\right)\right\}\right]+$ $\sum_{i=1}^{r}\varphi_i' \min\left[\Re e\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>\mu^{-2}$

Proof : To proof of equation (2.1) we first take the values of Fox's H-function. The general class of polynomials and M-series including all values from equation (1.1),(1.2), (1.3), (1.4) and (1.5) respectively. then express the H-function of multivariable in the form of Mellin-Barnes type contour integral. Now interchanging the order of summations and integrations which is permissible under the stated conditions, we acquire

$$\sum_{G!=0}^{\infty} \sum_{g=1}^{S} \sum_{S'=0}^{\infty} \sum_{L_1=0}^{[N_1/M_1]}, \dots, \sum_{L_\ell=0}^{[N_\ell/M_\ell]} \sum_{k_1+\dots+k_\ell}^{\alpha_1 k_1+\dots+\alpha_\ell k_\ell \le V} \frac{(-1)^G}{G!\,F_g} F(s')\xi(\eta_G)$$

$$(-V)_{\alpha_1 k_1+\dots+\alpha_\ell k_\ell} A(V;k_1,\dots,k_\ell) B[N_1,L_1;\dots;N_\ell,L_\ell] \frac{(-N_1)_{M_1,L_1}}{L_1},\dots,\frac{(-N_\ell)_{M_\ell,L_\ell}}{L_\ell}$$

$$\frac{W_1^{L_1+,K_1}}{L_1},\dots,\frac{W_\ell^{L_\ell+,K_\ell}}{L_\ell} \int_0^\infty k^{1-(\mu-\varphi\eta_G-\gamma s'-\sum_{i=1}^{\ell} s_i(L_i+,K_i)-\sum_{i=1}^{S}(\rho_i\lambda_i)} (R+ST+$$

$$RTk^2)^{(\mu-\varphi\eta_G-\gamma s'-\sum_{i=1}^{\ell} s_i(L_i+,K_i)-\sum_{i=1}^{S}(\rho_i\lambda_i)-3/2}$$

$$\frac{1}{(2\pi\omega)^s}\int_{\xi_1}\cdots\int_{\xi_r} \theta_i(\lambda_1,\dots,\lambda_s)\ \phi_1(\lambda_1),\dots,\phi_s(\lambda_s) z_1^{\lambda_1},\dots,z_s^{\lambda_s} dk$$

$$= \sum_{G!=0}^{\infty} \sum_{g=1}^{S} \sum_{S'=0}^{\infty} \sum_{L_1=0}^{[N_1/M_1]}, \dots, \sum_{L_\ell=0}^{[N_\ell/M_\ell]} \sum_{k_1+\dots+k_\ell}^{\alpha_1 k_1+\dots+\alpha_\ell k_\ell \le V} \frac{(-1)^G}{G!\,F_g} F(s')\,\xi(\eta_G)$$

$$(-V)_{\alpha_1 k_1+\dots+\alpha_\ell k_\ell} A(V;k_1,\dots,k_\ell) B[N_1,L_1;\dots;N_\ell,L_\ell] \frac{(-N_1)_{M_1,L_1}}{L_1},\dots,\frac{(-N_\ell)_{M_\ell,L_\ell}}{L_\ell} \frac{W_1^{L_1+,K_1}}{L_1},\dots,\frac{W_\ell^{L_\ell+,K_\ell}}{L_\ell} (R+Sk+$$

$$Tk^2)^{(\mu-\varphi\eta_G-\gamma s'-\sum_{i=1}^{\ell} s_i(L_i+,K_i)-\sum_{s=1}^{r}(\rho_s\lambda_s)-1}$$

$$\frac{1}{(2\pi\omega)^r}\int_{\zeta_1}\cdots\int_{\zeta_r} \frac{\Gamma\{1-(\mu-\varphi\eta_G-\gamma s'-\sum_{i=1}^{l} s_i(L_i+k_i)-\sum_{s=1}^{r}\rho_s\lambda_s)\}\Pi_{j=1}^{\Omega}\Gamma(1-a_j^{(i)}+\sum_{i=1}^{r}\alpha_j\lambda_i}{\Pi_{j=1}^{D}\Gamma\left(1-b_j^{(i)}+\sum_{i=1}^{r}\beta_j\lambda_i\right)\Gamma 1-(\mu-\varphi\eta_G-\gamma s'-\sum_{i=1}^{l} s_i(L_i+k_i)-\sum_{s=1}^{r}\rho_s\lambda_s-\frac{1}{2})}$$

$$\frac{\Pi_{j=1}^{m^{(i)}}\Gamma(d_j^{(i)}-\sum_{i=1}^{r}\delta^{(i)}\lambda_i\,\Pi_{j=1}^{n^{(i)}}\ \Gamma(1-b_j^{(i)}+\sum_{i=1}^{s}\beta_j^{(i)}\lambda_i}{\Pi_{j=\Omega+1}^{B}\Gamma(a_j\ \lambda_j-\sum_{s=1}^{r}\alpha_j\lambda_i\Pi_{j=m^{(i)}+1}^{q^{(i)}}\Gamma(1-d_j^{(i)}-\sum_{i=1}^{r}\delta^{(i)}\lambda_i\,\Pi_{j=n^{(i)}+1}^{p^{(i)}}\Gamma(b_j^{(i)}-\sum_{i=1}^{s}\beta_j^{(i)}\lambda_i)}$$

$$y_1^{\lambda_1},\dots,y_r^{\lambda_r}\,d\lambda_1,\dots,d\lambda_r$$

Now applying the definition of H-function of 'r' variable and using the formula (1.6). We arrived at the required result (2.1)

III SPECIAL CASES

Case -1 Now we reduced Ω = B = D = 0 and also using equation (1.1d), we get the following equation

$$\int_0^\infty k^{1-\mu}\ (R+Sk+Tk^2)^{\mu-3/2} H_{U,V}^{S,N}\left[\left(\frac{k}{(R+Sk+Tk^2)}\right)^\varphi \left| \begin{matrix}[k_P,K_P]\\ [f_Q,F_Q]\end{matrix}\right.\right]$$

$$S_{N_1,\dots,N_\ell}^{M_1,\dots,M_\ell}\left[w_1\left(\frac{k}{(R+Sk+Tk^2)}\right)^{s_1},\dots,w_\ell\left(\frac{k}{(R+Sk+Tk^2)}\right)^{s_\ell}\right] {}_pF_q\ \left[\left(\frac{k}{(R+Sk+Tk^2)}\right)^\gamma\right]$$

$$S_V^{\alpha_1,\dots,\alpha_\ell}\left[w_1\left(\frac{k}{(R+Sk+Tk^2)}\right)^{s_1},\dots,w_\ell\left(\frac{k}{(R+Sk+Tk^2)}\right)^{s_\ell}\right]\prod_{i=1}^s H_{p_i,q_i}^{m_i,n_i}$$

$$\left[y_i\left(\frac{k}{(R+Sk+Tk^2)}\right)^{\rho_i}\left| \begin{matrix}(c_j^{(i)},\gamma_j^{(i)})_{1,p_i}\\ (d_j^{(i)},\delta_j^{(i)})_{1,q_i}\end{matrix}\right.\right] dk$$

$$= \sqrt{\frac{\pi}{T}}\sum_{G!=0}^\infty \sum_{g=1}^S \sum_{S'=0}^\infty \sum_{L_1=0}^{[N_1/M_1]},\dots,\sum_{L_\ell=0}^{[N_\ell/M_\ell]}\sum_{k_1+\dots+k_\ell}^{\alpha_1k_1+\dots+\alpha_\ell k_\ell\le V}\frac{(-1)^G}{G!\,F_g}\ G(s')$$

$$\xi(\eta_G)(-V)_{\alpha_1k_1+\dots+\alpha_\ell k_\ell}A(V;k_1,\dots,k_\ell)B[N_1,L_1;\dots;N_\ell,L_\ell]\prod_{i=1}^{\ell}\frac{(-N_i)_{M_iL_i}}{L_i}\frac{W_i^{(L_i+,K_i)}}{L_i}$$

$$(S+2\sqrt{TR})^{(\mu-\varphi\eta_G-\gamma s'-\sum_{i=1}^{\ell}s_i(L_i+,K_i)-1)}H_{1,1:\,p_1,q_1;\dots;p_s,q_s}^{0,1:\,m_1,n_1;\dots;m_s,n_s}\left[\begin{matrix}y_1(S+2\sqrt{TR})^{-\rho_1}\\ \vdots\\ y_s(S+2\sqrt{TR})^{-\rho_s}\end{matrix}\right.$$

$$\left|\begin{matrix}(-\mu-\varphi\eta_G-\gamma s'-\sum_{i=1}^{\ell}s_i(L_i+,K_i);\rho_1,\dots,\rho_r),(a_j;\alpha_j^{(1)},\alpha_j^{(s)})_{1,p}:\dots;\dots;\dots\\ (b_j;\beta_j^{(1)},\beta_j^{(s)})_{1,q}(-\mu-\varphi\eta_G-\gamma s'-\sum_{i=1}^{\ell}s_i(L_i+K_i)-1/2;\rho_1,\dots,\rho_r):\dots;\dots;\dots\end{matrix}\right] \quad (3.1)$$

Case – II Using equation. (1.1f) and putting $\alpha^1,\dots,\alpha^{(r)}=\gamma^1,\dots,\gamma^{(r)}=\beta^1,\dots,\beta^{(r)}=\delta^1,\dots,\delta^{(r)}=\rho_1,\dots,\rho_r=\rho^1,\dots,\rho^{(r)}$ in (2.1) we find following result

$$\int_0^\infty k^{1-\mu}\ (R+Sk+Tk^2)^{\mu-3/2} H_{U,V}^{S,N}\left[\left(\frac{k}{(R+Sk+Tk^2)}\right)^\varphi \left| \begin{matrix}[k_P,K_P]\\ [f_Q,F_Q]\end{matrix}\right.\right]$$

$$S_{N_1,\dots,N_\ell}^{M_1,\dots,M_\ell}\left[w_1\left(\frac{k}{(R+Sk+Tk^2)}\right)^{s_1},\dots,w_\ell\left(\frac{k}{(R+Sk+Tk^2)}\right)^{s_\ell}\right] E_\gamma\ \left[\left(\frac{k}{(R+Sk+Tk^2)}\right)^\gamma\right]$$

$$S_V^{\alpha_1,\dots,\alpha_\ell}\left[w_1\left(\frac{k}{(R+Sk+Tk^2)}\right)^{s_1},\dots,w_\ell\left(\frac{k}{(R+Sk+Tk^2)}\right)^{s_\ell}\right]F_{1,1:\,(p^1,q^1),\dots,(p^r,q^r)}^{0,\Omega:\,(m^1,n^1),\dots,(m^r,n^r)}$$

$$\left[y_1^{\frac{1}{\rho_1}}\left(\frac{k}{(R+Sk+Tk^2)}\right)^{\rho_1},\dots,y_r^{\frac{1}{\rho_r}}\left(\frac{k}{(R+Sk+Tk^2)}\right)^{\rho_r}\left|\begin{matrix}(a);\ (c^{(1)}),\dots,(c^{(r)})\\ (b);\ (d^{(1)}),\dots,(d^{(r)})\end{matrix}\right.\right] dk$$

$$= \sqrt{\frac{\pi}{T}}\sum_{G!=0}^\infty \sum_{g=1}^S \sum_{S'=0}^\infty \sum_{L_1=0}^{[N_1/M_1]},\dots,\sum_{L_\ell=0}^{[N_\ell/M_\ell]}\sum_{k_1+\dots+k_\ell}^{\alpha_1k_1+\dots+\alpha_\ell k_\ell\le V}\frac{(-1)^G}{G!\,F_g}\ T(s')$$

$$\xi(\eta_G)(-V)_{\alpha_1k_1+\dots+\alpha_\ell k_\ell}A(V;k_1,\dots,k_\ell)B[N_1,L_1;\dots;N_\ell,L_\ell]\prod_{i=1}^{\ell}\frac{(-N_i)_{M_iR_i}}{L_i}\frac{W_i^{(L_i+,K_i)}}{L_i}$$

$$\left(S+2\sqrt{TR}\right)^{(\mu-\varphi\eta_G-\gamma s'-\sum_{i=1}^{\ell}s_i(L_i+K_i)-1)}\frac{\Gamma\{1-\mu-\varphi\eta_G-\gamma s'-\sum_{i=1}^{\ell}s_i(L_i+k_i)\}\rho}{\Gamma\{3/2-(\mu-\varphi\eta_G-\gamma s'-\sum_{i=1}^{\ell}s_i(L_i+k_i)\}}$$

$$E_{D+1:(q^1;\dots;q^{(r)})}^{B+1:(p^1;\dots;p^{(r)})},F_{B+1,D+1:(p^1,q^1);\dots;(p^r,q^r)}^{0,\Omega+1:(m^1,n^1);\dots;(m^r,n^r)}\begin{bmatrix}y_1^{-\frac{1}{\rho_1}}\left(S+2\sqrt{TR}\right)^{-1}\\ \vdots\\ y_r^{-\frac{1}{\rho_r}}\left(S+2\sqrt{TR}\right)^{-1}\end{bmatrix}$$

$$\left|\begin{matrix}\left(-\mu-\varphi\eta_G-\gamma s'-\sum_{i=1}^{\ell}s_i(L_i+K_i)\right),(a);\ c^{(1)},\dots,(c^{(r)})\\ (b);\ (d^{(1)}),\dots,(d^{(r)}):(-\mu-\varphi\eta_G-\gamma s'-\sum_{i=1}^{\ell}s_i(L_i+K_i)-1/2)\end{matrix}\right] \qquad (3.2)$$

Provided the $\Re e(R)>0,\Re e(S)>0,(T)>0\ and\ ,L_i>0\ (i=1,\dots,\ell),m^{(i)}$

$$(m^{(i)}+n^{(i)})(B+D+(p^{(i)}+q^{(i)})\beta\ |arg,y_i|<\left[(m^{(i)}+n^{(i)}-\frac{B}{2}-\frac{D}{2}-\frac{p^{(i)}}{2}-\frac{q^{(i)}}{2}\right]\pi$$

and $\beta\left[\min_{1\le j\le s}\left\{\Re e\left(\frac{f_j}{F_j}\right)\right\}\right]+\sum_{i=1}^{r}\left[\min_{1\le j\le m^{(i)}}\{\Re e(d^{(i)})\}\right]>\mu^{-2}$

Case – III Using equation. (1.1b) and putting $\Omega=B,m^{(i)}=1,n^{(i)}=n^{(i)}\ and\ n^{(i)}=p^{(i)}\ and\ q^{(i)}=q^{(i)}+1\ \ \forall i\in(1,\dots,r)$ the results in (2.1) reduced to the following integral transformation.

$$\int_0^\infty k^{1-\mu}\ (R+Sk+Tk^2)^{\ \mu-3/2}\ H_{U,V}^{S,N}\left[\left(\frac{k}{(R+Sk+Tk^2)}\right)^{\varphi}\left|\begin{matrix}[k_P,K_P]\\ [f_Q,F_Q]\end{matrix}\right.\right]$$

$$S_{N_1,\dots,N_\ell}^{M_1,\dots,M_\ell}\left[w_1\left(\frac{k}{(R+Sk+Tk^2)}\right)^{s_1},\dots,w_\ell\left(\frac{k}{(R+Sk+Tk^2)}\right)^{s_\ell}\right]\ {}_p^{\alpha}M_q^{\beta}\ \left[\left(\frac{k}{(R+Sk+Tk^2)}\right)^{\gamma}\right]$$

$$S_V^{\alpha_1,\dots,\alpha_\ell}\left[w_1\left(\frac{k}{(R+Sk+Tk^2)}\right)^{s_1},\dots,w_\ell\left(\frac{k}{(R+Sk+Tk^2)}\right)^{s_\ell}\right]E_{D:\,q^{(1)};\dots;q^{(r)}}^{B:\,p^{(1)};\dots;p^{(r)}}$$

$$\left[y_1\left(\frac{k}{(R+Sk+Tk^2)}\right)^{\rho_1},\dots,y_r\left(\frac{k}{(R+Sk+Tk^2)}\right)^{\rho_r}\left|\begin{matrix}\left(1-(a_j);\ \alpha_j^{(1)},\dots,\alpha_j^{(r)}\right)_{1,p}:\\ \left(1-(b_j);\ \beta_j^{(1)},\dots,\beta_j^{(r)}\right)_{1,q}:\end{matrix}\right.\right.$$

$$\left.\begin{matrix}\left(1-(c_j^{(1)}),\gamma_j^{(1)},\right)_{1,p_1};\dots;\left(1-(c_j^{(1)}),\gamma_j^{(1)},\right)_{1,p_r}\\ \left(1-(d_j^{(1)}),\delta_j^{(1)},\right)_{1,p_1};\dots;\left(1-(d_j^{(r)}),\delta_j^{(r)},\right)_{1,p_r}\end{matrix}\right]dk$$

$$=\sqrt{\frac{\pi}{T}}\sum_{G!=0}^{\infty}\sum_{g=1}^{S}\sum_{S'=0}^{\infty}\sum_{L_1=0}^{[N_1/M_1]},\dots,\sum_{L_\ell=0}^{[N_\ell/M_\ell]}\sum_{k_1+\dots+k_\ell}^{\alpha_1k_1+\dots+\alpha_\ell k_\ell\le V}\frac{(-1)^G}{G!\,F_g}\ L(s')$$

$$\xi(\eta_G)(-V)_{\alpha_1k_1+\dots+\alpha_\ell k_\ell}A(V;k_1,\dots,k_\ell)B[N_1,L_1;\dots;N_\ell,L_\ell]\prod_{i=1}^{\ell}\frac{(-N_i)_{M_iL_i}}{L_i}\frac{W_i^{(L_i+K_i)}}{k_i}$$

$$\left(S+2\sqrt{TR}\right)^{(\mu-\varphi\eta_G-\gamma s'-\sum_{i=1}^{\ell}s_i(L_i+K_i)-1)}\frac{\Gamma\{1-\mu+\varphi\eta_G+\gamma s'+\sum_{i=1}^{\ell}s_i(L_i+k_i)\}}{\Gamma\{\frac{3}{2}-\mu+\varphi\eta_G+\gamma s'+\sum_{i=1}^{\ell}s_i(L_i+k_i)\}}$$

$$E_{D+1:(q^1;\dots;q^{(r)})}^{B+1:(p^1;\dots;p^{(r)})}\begin{bmatrix}y_1^{-\frac{1}{\rho_1}}\left(S+2\sqrt{TR}\right)^{-1}\\ \vdots\\ y_r^{-\frac{1}{\rho_r}}\left(S+2\sqrt{TR}\right)^{-1}\end{bmatrix}\left|\begin{matrix}\left[\left(1-\mu+\varphi\eta_G+\gamma s'+\sum_{i=1}^{\ell}s_i(L_i+K_i)\right):\rho_1,\dots,\rho_r\right]\\ \left(1-(b_j);\ \beta_j^{(1)},\dots,\beta_j^{(r)}\right)_{1,q}:\left(\frac{3}{2}-\mu+\varphi\eta_G+\gamma s'\right.\end{matrix}\right.$$

$$,(1-(a_j);\ \alpha_j^{(1)},\dots,\alpha_j^{(r)})_{1,p}(1-(c_j^{(1)}),\gamma_j^{(1)},)_{1,p_1};\dots;\ (1-(c_j^{(1)}),\gamma_j^{(1)},)_{1,p_r}$$
$$+\sum_{i=1}^{\ell} s_i(L_i+K_i):\rho_1,\dots,\rho_r):(1-(d_j^{(1)}),\delta_j^{(1)},)_{1,q_1};\dots;(1-(d_j^{(r)}),\delta_j^{(r)},)_{1,q_r}\Bigg]dk \quad (3.3)$$

Provided the $\Re e(R) > 0, \Re e(S) > 0, (T) > 0$ hence the series on the right side exists.

Case - IV If r = 1 and M_i, $N_i \to 0$, (i=2,…,s'), $V \to 0$, the product in (2.1) become to known result after a slight modification recently consequent by Gupta and Jain[7].

Case-V Reduced $N_i \to 0$, (i = 1,…, ℓ), $V \to 0$, T = 0, R = 1, then the result in (2.1) become to the known results after a slight simplification obtained by Goyal and Mathur [6] .

Case-VI Taking $V \to 0$, the result in (2.1) proceeding to the known result given in [2.1] after a little simplification introduced by chaurasiya and Shekhawat [1]

IV. CONCLUSION

The multivariable H-function of and M-series described with various polynomials in research paper. This work relatively basic in nature, as a result of specifying of the parameters on this function, we would find another special function like as Majer's G-function, Mac-robent's, Bessel function, Wright's function, Fox's H-function, generalized hypergeometric function and furthermore on the specializing the parameters of this function obtain various special cases.

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