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Solution of Real-life optimization problem with Fixed Point Theory

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Abstract

The main object of this paper is to find application of linear programming with fixed point theory in Assignment problem for optimization and for this to assign a particular teacher to a particular subject to minimize the total cost of a particular school or institute with constraints that each teacher can teach every subject and each subject can be teach by every teacher but with different teaching costs.
Keywords: Linear programming model; Constraints; Objective function; Assignment problem; Cost minimization; soft set, fixed point

1. Introduction and preliminaries

The fixed-point theory plays an important role to solve the real-life engineering and problem related to optimization. Maximum of our traditional tools for formal modelling and computing are crisp. There are many complicated problems in economics and engineering that involve data which are not always all crisp. The classical method can not be used due to different type of uncertainty. Molodtsov [2] given the theory of soft set as the tool for solving uncertainty problems which is also useful for decision making problems with certain parameters. Th theory is also useful for optimization techniques for decision making. The detail about soft theory for decision making problems can be viewed in [8-10] The basic theorem of fixed point that is known as Banach's contraction principle was given by S. Banach[1] has many applications to solve optimization problems. The fixed point techniques for optimization can be viewed in [6-7].
Convex minimization problem is one of the central problems in nonlinear analysis through fixed point. Iterative algorithms are suggested by [5]. One of the convex minimization problem over fixed point was taken by [6] as,
Let g: H⇢ R is strictly convex and T is a self-mapping on H such that fixed point set, $Fix\,(T) = \{x \in H: T(x) = x\}$, then to find $x^* \in Fix\,(T): g(x^*) = \underbrace{min}_{x\in Fix\,(T)}\ g(x)$
The aim of present research paper is to the find optimization solution of assignment problem motivating by [6]. The assignment problem is a part of linear programming problems which is a special type of transportation problem in which available resources and demand both are equal to1. Using assignment algorithm, the administrative department of any company attain their required result or allocation by reducing the total cost or total time of the company to get maximum profit. Mostly the administrative department do not apply the effective methods for initial assignment for a group of workers and therefore there is large amount of waste of time and money and increase the overall cost of the company.
In assignment problem we have to minimize total time or cost which required to complete the jobs by assigning some precise jobs to correct workers or machines. Linear programming is an efficient tool for solving assignment problems which contain equal number of jobs and workers or machines for optimization to complete the jobs.
The mathematical form of assignment problems are as:

Minimize the total cost

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij}\, T_{ij}$$

Where $T_{ij} = 1\ or\ 0 \quad =$

Subject to the condition

$$(i) \sum_{i=1}^{n} T_{ij} = 1, j = 1,2,\dots,n$$

$$(ii) \sum_{j=1}^{n} T_{ij} = 1, i = 1,2,\dots,n$$

Fathallah Fadhil Khalaf Al-Abdhulhameed [4] used linear programming for solution of assignment problem. Elsidding Idress Mohamed et al. [3] discussed Hungarian method for solving assignment problem and many other authors used linear programming for maximum profit and for job scheduling and for solution of assignment problems for minimum cost or time.

The main intension of this paper is to assign a particular teacher to a particular subject to find minimum cost for a particular school or institute.

The general form of linear programming problem is

Optimize

$Z = C_{11}T_{11} + C_{12}T_{12} + \dots + C_{mn}T_{mn}$ (objective function) .....(i)

subject to the constraints

$$\left.\begin{array}{l} a_{11}T_{11} + a_{12}T_{12} + \dots + a_{1n}T_{1n} (\le, =, \ge) b_1 \\ a_{21}T_{21} + a_{22}T_{22} + \dots + a_{2n}T_{2n} (\le, =, \ge) b_2 \\ \dots\dots \quad \dots\dots \quad \dots\dots \quad \dots\dots \\ \dots\dots \quad \dots\dots \quad \dots\dots \quad \dots\dots \\ a_{m1}T_{m1} + a_{m2}T_{m2} + \dots + a_{mn}T_{mn} (\le, =, \ge) b_m \end{array}\right\} \quad \dots (ii)$$

and non-negative restrictions $T_{ij} \ge 0, \quad i = 1, 2...m,\ j = 1, 2...n$

Where aij's, bi's and Cj's are constants and T_{ij}'s are variables.

In the conditions given by (ii) there may be any of the three signs $\le, =, \ge$.

The standard form of the linear programming problem can be written as follows:

Optimize

$Z = C_{11}T_{11} + C_{12}T_{12} + \dots + C_{mn}T_{mn} + 0.S_1 + 0.S_2 + \dots + 0.S_m$ (objective function) … ..(iii)

subject to the constraints

$$\left.\begin{array}{l} a_{11}T_{11} + a_{12}T_{12} + \dots + a_{1n}T_{1n} + S_1 = b_1 \\ a_{21}T_{21} + a_{22}T_{22} + \dots + a_{2n}T_{2n} + S_2 = b_2 \\ \dots\dots \quad \dots\dots \quad \dots\dots \quad \dots\dots \\ \dots\dots \quad \dots\dots \quad \dots\dots \quad \dots\dots \\ a_{m1}T_{m1} + a_{m2}T_{m2} + \dots + a_{mn}T_{mn} + S_m = b_m \end{array}\right\} \quad \dots (iv)$$

and non-negative restrictions $T_{ij} \ge 0, \quad S_i \ge 0,\ j = 1, 2...n, i = 1, 2...m$

Where aij's, bi's and Cj's are constants and T_{ij}'s and Si's are variables.

2. Assumption for problem

(a) It is assumed that the number of teachers and subjects are fixed and equal.
(b) It is assumed that only one teacher can teach only one subject and only one subject can be teach by only one teacher.
(c) It is assumed that all relation are linear.

3. Data presentation and analysis

The Cost matrix of the problem is given as follows-

Teachers	Cost (In thousands)				
	Subject (1)	Subject (2)	Subject (3)	Subject (4)	Subject (5)
T_1	15	33	30	36	12
T_2	6	12	18	9	15
T_3	9	36	15	42	18
T_4	18	42	12	33	21
T_5	21	27	24	36	15

4. Model formulation

Let T_{11} denote the assignment of teacher T_1 to subject 1.
Let T_{12} denote the assignment of teacher T_1 to subject 2.
Let T_{13} denote the assignment of teacher T_1 to subject 3.
Let T_{14} denote the assignment of teacher T_1 to subject 4.
Let T_{15} denote the assignment of teacher T_1 to subject 5.
Let T_{21} denote the assignment of teacher T_2 to subject 1.
Let T_{22} denote the assignment of teacher T_2 to subject 2.
Let T_{23} denote the assignment of teacher T_2 to subject 3.
Let T_{24} denote the assignment of teacher T_2 to subject 4.
Let T_{25} denote the assignment of teacher T_2 to subject 5.
Let T_{31} denote the assignment of teacher T_3 to subject 1.
Let T_{32} denote the assignment of teacher T_3 to subject 2.
Let T_{33} denote the assignment of teacher T_3 to subject 3.
Let T_{34} denote the assignment of teacher T_3 to subject 4.
Let T_{35} denote the assignment of teacher T_3 to subject 5.
Let T_{41} denote the assignment of teacher T_4 to subject 1.

Let T_{42} denote the assignment of teacher T_4 to subject 2.
Let T_{43} denote the assignment of teacher T_4 to subject 3.
Let T_{44} denote the assignment of teacher T_4 to subject 4.
Let T_{45} denote the assignment of teacher T_4 to subject 5.
Let T_{51} denote the assignment of teacher T_5 to subject 1.
Let T_{52} denote the assignment of teacher T_5 to subject 2.
Let T_{53} denote the assignment of teacher T_5 to subject 3.
Let T_{54} denote the assignment of teacher T_5 to subject 4.
Let T_{55} denote the assignment of teacher T_5 to subject 5.
Let Z is total cost to be minimize.
The mathematical form of above data is
Minimize

$$Z = 15T_{11} + 33T_{12} + 30T_{13} + 36T_{14} + 12T_{15} + 6T_{21} + 12T_{22} + 18T_{23} + 9T_{24} + 15T_{25} + 9T_{31} + 36T_{32} + 15T_{33}$$
$$+ 42T_{34} + 18T_{35} + 18T_{41} + 42T_{42} + 12T_{43} + 33T_{44} + 21T_{45} + 21T_{51} + 27T_{52} + 24T_{53} + 36T_{54} + 15T_{55}$$

Subject to

$$T_{11} + T_{12} + T_{13} + T_{14} + T_{15} = 1$$
$$T_{21} + T_{22} + T_{23} + T_{24} + T_{25} = 1$$
$$T_{31} + T_{32} + T_{33} + T_{34} + T_{35} = 1$$
$$T_{41} + T_{42} + T_{43} + T_{44} + T_{45} = 1$$
$$T_{51} + T_{52} + T_{53} + T_{54} + T_{55} = 1$$
$$T_{11} + T_{21} + T_{31} + T_{41} + T_{51} = 1$$
$$T_{12} + T_{22} + T_{32} + T_{42} + T_{52} = 1$$
$$T_{13} + T_{23} + T_{33} + T_{43} + T_{53} = 1$$
$$T_{14} + T_{24} + T_{34} + T_{44} + T_{54} = 1$$
$$T_{15} + T_{25} + T_{35} + T_{45} + T_{55} = 1$$

and $T_{ij} \geq 0, i \,\&\, j = 1, 2, 3, 4, 5$

Using Lingo software for solving above linear programming problem, we get an optimal solution as

$$T_{11} = 0, T_{12} = 0, T_{13} = 0, T_{14} = 0, T_{15} = 1 \quad T_{21} = 0, T_{22} = 0, T_{23} = 0, T_{24} = 1, T_{25} = 0$$
$$T_{31} = 1, T_{32} = 0, T_{33} = 0, T_{34} = 0, T_{35} = 0 \quad T_{41} = 0, T_{42} = 0, T_{43} = 1, T_{44} = 0, T_{45} = 0$$
$$T_{51} = 0, T_{52} = 1, T_{53} = 0, T_{54} = 0, T_{55} = 0$$

Minimize Z = 69

5. Interpretation of result

The optimal solution of the above assignment problem based on assumed data specified that the minimum value of Z is 69 and value of $T_{15} = 1$, $T_{24} = 1$, $T_{31} = 1$, $T_{43} = 1$, $T_{52} = 1$ and value of other variables zero.

6. Conclusion

From the above discussion it is concluded that the minimum cost is 69 thousand by assigning of teacher T1 to subject 5, assigning of teacher T2 to subject 4, assigning of teacher T3 to subject 1, assigning of teacher T4 to subject 3, assigning of teacher T5 to subject 2.

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