Abstract

Digital cameras sample scenes using a color filter array of mosaic pattern (e.g., the Bayer pattern). The demosaicking of the color samples is critical to the image quality. This paper presents a new color demosaicking technique of optimal directional filtering of the green–red and green–blue difference signals. Under the assumption that the primary difference signals (PDS) between the green and red/blue channels are low pass, the missing green samples are adaptively estimated in both horizontal and vertical directions by the linear minimum mean square-error estimation (LMMSE) technique. These directional estimates are then optimally fused to further improve the green estimates. Finally, guided by the demosaicked full-resolution green channel, the other two color channels are reconstructed from the LMMSE filtered and fused PDS. The experimental results show that the presented color demosaicking technique outperforms the existing methods both in PSNR measure and visual perception.

Keywords:—Color demosaicking, Bayer color filter array (CFA), directional filtering, linear minimum mean square-error estimation (LMMSE).

I. Introduction

Most digital cameras capture an image with a single sensor array. At each pixel, only one of the three primary colors (red, green, and blue) is sampled. Fig. 1 shows the commonly used Bayer color filter array (CFA) [5]. In order to reconstruct a full color image, the missing color samples need to be interpolated by a process called color demosaicking. The quality of reconstructed color images depends on the image contents and the employed demosaicking algorithms [15].

The early demosaicking methods include nearest neighbor replication, bilinear interpolation and cubic B-spline interpolation [1], [10], [15]. These methods can be simply implemented, but they suffer from many artifacts such as blocking, blurring and zipper effect at edges. Under the assumption that images have a slowly varying hue, the smooth hue transition (SHT) methods [1], [6], [20] interpolate the luminance (green) channel and chrominance (red and blue) channels differently.

![Bayer pattern](image)

**Fig. 1.** Bayer pattern

Many adaptive demosaicking methods try to avoid interpolating across edges [2], [3], [7], [11], [12], [16], [18]. At each pixel, the gradient is estimated, and the color interpolation is carried out directionally based on the estimated gradient. Directional filtering is the most popular approach to color demosaicking that produces competitive results in the literature. The best-known directional interpolation scheme is, perhaps, the second-order Laplacian filter proposed by Hamilton and Adams [2], [3], [11]. They used the second-order gradients of blue and red channels as the correction terms to interpolate the green channel. The smaller of the two second-order gradients in the horizontal and vertical directions is added to the average of the green samples along the chosen direction. Once the green samples are filled, the red and blue samples are interpolated similarly with the correction of the second-order gradients of the green channel. Chang et al. [7] proposed a more complicated gradient-based demosaicking scheme. They computed a set of gradients in different directions in the 5x5 neighborhood centered at the pixel to be interpolated. A subset of these gradients is selected by adaptive thresholding. At last, the missing samples are estimated from the known samples located along the selected gradients. Recently, Ramanath and Snyder [18] proposed a bilateral filtering based scheme to denoise, sharpen and demosaick the image simultaneously. Alleysson et al. [4] treated a color pixel as the sum of luminance and chrominance, and reconstructed the image by selecting the luminance and chrominance components in Fourier domain.

Another class of color demosaicking techniques is iterative schemes, which can also be combined with gradient-based methods. Kimmel developed a two-step iterative demosaicking process consisting of a reconstruction step and an enhancement step [13].
red and blue samples are then corrected iteratively by the ratio rule. Finally, an inverse color diffusion process is applied to the whole image for enhancement. Another iterative demosaicking scheme was proposed by Gunturk et al. [9]. The color samples were estimated by bilinear or other demosaicking methods. The initial estimates were projected onto so-called constraint sets, and then iteratively improved in the wavelet domain by updating the high-frequency details of the red and blue channels according to the green channel. Other demosaicking methods were also proposed, such as minimum mean square-error (MSE) estimation [19], pattern matching [21], and median filtering [8]. In all color demosaicking techniques, gradient analysis plays a central role in reconstructing sharp edges. However, the gradient estimate may not be robust when the input signal exceeds the Nyquist frequency. This is the main cause of color artifacts in demosaicked images. The challenge is to use statistically valid constraints to overcome the limit of Nyquist frequency. A common practice in color demosaicking is to exploit the correlation between the color channels. Since the three color channels of a natural image are highly correlated, the difference signal between the green channel and the red or blue channel constitutes a smooth (low-pass) process. Furthermore, we observe that this color difference signal is largely uncorrelated to the interpolation errors of gradient-guided color demosaicking methods, and exhibits a band-pass behavior. Based on these observations, we propose to estimate the color difference signals by linear minimum mean square-error estimation (LMMSE) technique, which can be shown to be a good approximation to optimal estimation in MSE sense. We obtain the LMMSE estimates in both horizontal and vertical directions, and then fuse the two estimates optimally to remove the demosaicking noise. color demosaicking technique significantly outperforms the state-of-the-art methods both in PSNR measure and visual perception. This paper is structured as follows. In Section II, we introduce the notions of primary difference signal (PDS) and the directional demosaicking noises. Section III presents the LMMSE technique of estimating primary difference signals in both horizontal and vertical directions. Section IV describes how these two directional estimates can be optimally fused into a more robust estimate. Then, in Section V, the chrominance channels are interpolated based on the estimated PDS and luminance channel. Section VI gives the experimental results, and Section VII concludes.

II. Primary Difference Signal And Directional Demosaicking Noise

In order to recover high-frequency features beyond the designed Nyquist frequency of the CFA, a color demosaicking algorithm has to rely on some additional statistical properties or constraint(s) about the input color signals. A commonly exploited property is the correlation between the sampled primary color channels: red, green, and blue. In order to utilize this property in demosaicking, let us examine the relationships between the green and red channels, and between the green and blue channels. There are multiple reasons for why the green channel plays a key role in our estimation of missing color samples. First, the green channel has twice as many samples as the other two channels in the ubiquitous Bayer mosaic pattern, which is by far the prevailing CCD sensor design. Second, the sensitivity of the human visual system peaks at the green wavelength. Third, the green

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TABLE I: Correlation Coefficients of All Pairs of Primary Color Channels. $c_{gg}$ is the Correlation Coefficient of Green and Red Channels, $c_{rg}$ is the Correlation Coefficient of Green and Blue Channels, and $c_{bb}$ is the Correlation Coefficient of Red and Blue Channels.
channels measured over a set of 18 color test images shown in Fig. 2 and shows that the green–red and green–blue correlations are consistently greater than the red–blue correlation.

In the color demosaicking literature, two assumptions were made on green–red and green–blue relations: equal ratio [11], [6], [13], [20] and equal difference [2], [3], [7], [11]. The former assumption holds for mosaic CCD data prior to gamma correction, while the latter assumption is closer to the reality for gamma corrected mosaic CCD data. In this paper, we make a slightly more relaxed assumption that the difference images between the green and red channels and between the green and blue channels are low-pass signals, which are referred to the sequel as primary difference signals (PDS) and denoted by

$$\Delta_{g,r}(n) = G_n - R_n; \quad \Delta_{g,b}(n) = G_n - B_n$$ (2.1)

where is the position index of the pixels. The term is used because (2.1) represents two images whose pixel values are differences between corresponding green and red/blue samples. Under the assumption that $\Delta_{g,r}$ and $\Delta_{g,b}$ are smooth signals (some power spectrum density functions of $\Delta_{g,r}$ and $\Delta_{g,b}$ are plotted in Section III to support this assumption), we demosaic the green channel first and then the other two channels. The quality of final full-color reconstruction largely hinges on the estimation accuracy of the missing green samples in the Bayer pattern, because the reconstructed green channel has an anchor effect on subsequent steps of demosaicking the red and blue channels as we will see in Section V. We estimate PDS $\Delta_{g,r}$ and $\Delta_{g,b}$ rather than individual color channels directly because

The results for the symmetric case of estimating the missing green values at the blue sampling positions of the Bayer pattern can be derived in the same way. We denote the red sample at the center of the window as $R_0$. Its interlaced red and green neighbors in horizontal direction are labeled as $R^h_i, i \in \{..., -4, -2, 2, 4, ..., \}$, and $G^h_i, i \in \{..., -3, -1, 1, 3, ..., \}$, respectively; similarly, the red and green neighbors of in vertical direction are $R^v_j, j \in \{..., -4, -2, 2, 4, ..., \}$, and $G^v_j, j \in \{..., -3, -1, 1, 3, ..., \}$ respectively. The sample $R_0$ at the intersection can be taken as $R^h_0$ or $R^v_0$ at will.

To get some coarse measurements of PDS $\Delta_{g,r}$ and $\Delta_{g,b}$, we first interpolate the missing green samples at red and blue pixels and then interpolate the missing red and blue samples at green samples. Any of the existed interpolation methods for color demosaicking [2]–[4], [6]–[9], [11]–[13], [16], [18] may be used. We adopt the second-order Laplacian interpolation filter for its easy implementation and good performance (but we stress that the following development is independent of the interpolation methods). For any red original sample $R^h_i$ or $R^v_j$,

$$\hat{G}^h_i = \frac{1}{2} \left( G^h_{i-1} + G^h_{i+1} \right) + \frac{1}{4} \left( 2 \cdot R^h_{i-1} - R^h_{i-2} - R^h_{i+1} \right)$$

$$\hat{G}^v_j = \frac{1}{2} \left( G^v_{j-1} + G^v_{j+1} \right) + \frac{1}{4} \left( 2 \cdot R^v_{j-1} - R^v_{j-2} - R^v_{j+1} \right).$$

Similarly, for any original green sample $G^h_i$ or $G^v_j$, the corresponding missing red sample is interpolated as

$$\hat{R}^h_i = \frac{1}{2} \left( R^h_{i-1} + R^h_{i+1} \right) + \frac{1}{4} \left( 2 \cdot G^h_i - G^h_{i-2} - G^h_{i+2} \right)$$

$$\hat{R}^v_j = \frac{1}{2} \left( R^v_{j-1} + R^v_{j+1} \right) + \frac{1}{4} \left( 2 \cdot G^v_j - G^v_{j-2} - G^v_{j+2} \right).$$

Using the interpolated missing green and red values, we obtain two estimates of the random process $\Delta_{g,r}$ in horizontal and vertical directions, respectively

$$\hat{\Delta}_{g,r}^h(i) = \begin{cases} \frac{G^h_i - \hat{R}^h_i}{\hat{R}^h_i - \hat{R}^h_{i-1}}, & \text{G is interpolated} \\ \frac{G^h_i - \hat{R}^h_i}{\hat{R}^h_i - \hat{R}^h_{i+1}}, & \text{R is interpolated} \end{cases}$$

$$\hat{\Delta}_{g,r}^v(i) = \begin{cases} \frac{G^v_i - \hat{R}^v_i}{\hat{R}^v_i - \hat{R}^v_{i-1}}, & \text{G is interpolated} \\ \frac{G^v_i - \hat{R}^v_i}{\hat{R}^v_i - \hat{R}^v_{i+1}}, & \text{R is interpolated} \end{cases}.$$
Fig. (2). Test images used in this paper
The estimation errors associated with $\hat{\Delta}_{g,r}^h$ and $\hat{\Delta}_{g,r}^v$ are

$$
\begin{align*}
\epsilon_{g,r}^h &= \Delta_{g,r} - \hat{\Delta}_{g,r}^h, \\
\epsilon_{g,r}^v &= \Delta_{g,r} - \hat{\Delta}_{g,r}^v.
\end{align*}
$$

We regard $\hat{\Delta}_{g,r}^h$ and $\hat{\Delta}_{g,r}^v$ to be two observations of $\Delta_{g,r}$ and accordingly, $\epsilon_{g,r}^h$ and $\epsilon_{g,r}^v$ to be the corresponding directional demosaicking noises, and rewrite (2.7) as to be the corresponding directional demosaicking noises, and rewrite (2.7) as

$$
\begin{align*}
\hat{\Delta}_{g,r}^h &= \Delta_{g,r} - \epsilon_{g,r}^h, \\
\hat{\Delta}_{g,r}^v &= \Delta_{g,r} - \epsilon_{g,r}^v.
\end{align*}
$$

Now, the task is to obtain an optimal estimate of $\Delta_{g,r}$ from the two observation sequences $\{\hat{\Delta}_{g,r}^h\}$ and $\{\hat{\Delta}_{g,r}^v\}$, and then derive the missing green values. The estimation algorithm will be developed in Section III. To simplify the notations, we denote by the true PDS signal $\Delta_{g,r}$ and by the associated observation $\hat{\Delta}_{g,r}^h$ or $\hat{\Delta}_{g,r}^v$, and by U, the associated demosaicking noise $\epsilon_{g,r}^h$ or $\epsilon_{g,r}^v$, namely

$$y(n) = x(n) + v(n).$$

$$\hat{x} = E[x/y] = \int x p(x/y) dx.$$  \hspace{1cm} (2.10)

However, the MMSE estimation is very difficult, if possible at all, because $p(x/y)$ is seldom known in practice. Instead, we use the LMMSE technique to estimate $\Delta_{g,r}$ from $\hat{x}$, which is a good approximation to MMSE but more amenable to efficient implementation. Particularly, if $x(n)$ and $v(n)$ are locally Gaussian processes (a reasonable assumption for natural image signals), then the spatially adaptive LMMSE developed in Section III will be equivalent to MMSE [14].

The LMMSE of $x$ is computed as

$$\hat{x} = E[x] + \frac{\text{Cov}(x,y)}{\text{Var}(y)} (y - E[y]).$$  \hspace{1cm} (2.11)

Empirically, we found that the demosaicking noises $\epsilon_{g,r}^h$ or $\epsilon_{g,r}^v$ are zero-mean random processes, and they are almost uncorrelated with $\Delta_{g,r}$. This can be seen in Table II that lists the correlation coefficient $c_h$ between $\epsilon_{g,r}^h$ and $\Delta_{g,r}$ and the correlation coefficient $c_v$ between $\epsilon_{g,r}^v$ and $\Delta_{g,r}$ for the test images in Fig. 2 (the mosaic data of them are simulated by subsampling with the CFA of the Bayer pattern), in which $c_h$ and $c_v$, are, in fact, very close to zero. Consequently, we can simplify (2.11) to

$$\hat{x} = \mu_x + \frac{\sigma_x^2}{\sigma_x^2 + \sigma_y^2} (y - \mu_x).$$  \hspace{1cm} (2.12)

where $\mu_x = E[x], \sigma_x^2 = \text{Var}(x), \sigma_y^2 = \text{Var}(v)$. Symmetrically, we can define the difference signal $\Delta_{g,b}$ between the green and blue channels, and its two estimates $\hat{\Delta}_{g,b}^h$ and $\hat{\Delta}_{g,b}^v$, in horizontal and vertical directions. The corresponding estimation errors $\epsilon_{g,b}^h$ and $\epsilon_{g,b}^v$ have the same properties as those of $\epsilon_{g,r}^h$ and $\epsilon_{g,r}^v$. Theorem 2.
III. Directional LMMSE Of Primary Difference Signals

Having the knowledge of the statistical properties of the directional demosaicking noises $\varepsilon^h_{\theta,y}$ and $\varepsilon^v_{\theta,y}$, we now proceed to estimate PDS $\Delta_{\theta,y}$ by (2.12). To compute the LMMSE estimate, we need to estimate $\hat{x}(n)$, the three parameters $\mu_x$, $\sigma_x$, and $\sigma_v$, from observation data $y(n)$, and, in order to make the estimate $\hat{x}(n)$ spatially adaptive, these parameters should be estimated locally in the neighborhood of $y(n)$. We rely on the property of $x(n)$ that it is a low-pass process and $x(n)$ is a band-pass process to differentiate $x$ from $x$ in $x$. To verify this property, let us examine the power spectrum density functions of $x(n)$ and $x(n)$. The power spectrum density function of a time series $S$ is defined as the Fourier transform of the auto-correlation function of $S$.

$$f_x(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} f_x(k)e^{-j\omega k}$$  \hspace{1cm} (3.1)

where the sequence $f_x(k)$ is the auto-correlation function of $S$:

$$f_x(k) = E[S(n)S(n-k)]$$  \hspace{1cm} (3.2)

Since $f_x(k) = f_x(-k)$, (3.1) can be written as

$$f_x(\omega) = \frac{1}{2\pi} \left(f_x(0) + 2\sum_{k=1}^{\infty} f_x(k)\cos(k\omega)\right)$$  \hspace{1cm} (3.3)

$$y(n) = (y*h)(n) = \sum_{l=-\infty}^{\infty} y(n-k)\cdot h(k)$$  \hspace{1cm} (3.4)

$$y(n) = (y*k)(n) = \sum_{l=-\infty}^{\infty} y(n-k)\cdot h(k)$$  \hspace{1cm} (3.4)

where "*" is the convolution operator. In this paper, we set $\{h(\omega)\}$ to be the Gaussian smooth filter, whose coefficients are

$$h(k) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{k^2}{2\sigma^2}}$$  \hspace{1cm} (3.5)

where parameter $\sigma$ controls the shape of the filter response. Assuming that the random process $x(n)$ is ergodic and

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![Fig. 4. (a–f) Power spectrum functions of the green-red difference signals in horizontal direction for the first four images in Fig. 2. The power spectrum functions in vertical direction are similar. It is clear that PDS is a low frequency dominated process.](image-url)
The power spectrum density functions of $\mathcal{X}$ and $\mathcal{U}$ are plotted in Figs. 4 and 5 for some typical natural images. In Fig. 4, the power spectrum of $\mathcal{X}$ for the first four images in Fig. 2 are replotted, and, in Fig. 5, the corresponding power spectrum of $\mathcal{U}$ are illustrated. Obviously, the power of $\mathcal{X}$ is concentrated in low frequency band, whereas the power of $\mathcal{U}$ is spread in relatively high-frequency bands. Since $\mathcal{X}$ and $\mathcal{U}$ have distinct power spectrum, passing $\mathcal{U}$ through a low-pass filter can remove the noises effectively. Denoted by $\{h(k)\}$, the response sequence of a low-pass filter, we have stationary, its mean value $\mu_x(n)$ can be estimated by the neighboring data of $\mathcal{U}(n)$. The low-pass filter output $\mathcal{Y}(n)$ is a weighted average of $\mathcal{U}(n)$ and its neighbors, and it is much closer to $\mathcal{X}(n)$ than $\mathcal{U}(n)$. Denoted by $\mathcal{Y}_L = [y_x(n-L) \cdots y_x(n) \cdots y_x(n+L)]$ (3.6) the $2L+1$-dimensional vector centered at $y_x(n)$, we estimate $\mu_x(n)$ as

$$\mu_x(n) = \frac{1}{2L+1} \sum_{k=-L}^{L} Y_x^*(k)$$

and then estimate $\sigma_x^2(n)$, the variance of $\mathcal{X}(n)$, by

$$\sigma_x^2(n) = \frac{1}{2L+1} \sum_{k=-L}^{L} (Y_x^*(k) - \mu_x(n))^2.$$  

(3.8)

$$\sigma_y^2(n) = \frac{1}{2L+1} \sum_{k=-L}^{L} (Y^*(k) - Y_x(k))^2.$$  

(3.10)

For each sample $\mathcal{X}(n)$ to be estimated, the corresponding parameters $\mu_x(n)$, $\sigma_x^2(n)$ and are computed and substituted into (2.12) to yield $\hat{x}(n)$, the nearly LMMSE estimate of $\mathcal{X}(n)$. Let $\tilde{x}(n)$ be the estimation error of $\hat{x}(n)$, $x(n) = x(n) - \hat{x}(n)$, the variance of $\tilde{x}(n)$ is
Denoted by
\[ Y_n = [y(n-L) \ldots y(n) \ldots y(n+L)] \quad (39) \]
the $2L+1$-dimensional vector centered at $u(n)$. Since $Y_n$ is an approximation of $x(n)$, it follows $y(n) - u(n)$, that is an approximation of $v(n)$; thus, we can estimate $\sigma^2_v(n)$, the variance of $v(n)$, by
\[ \sigma^2_v(n) = E[y^2(n)] = \sigma^2_v(n) - \frac{\sigma^2_v(n)}{\sigma^2_v(n) + \sigma^2_v(n)} \quad (31) \]

IV. Optimal Fusion Of The Directional LMMSE Estimates

Using the scheme developed in the previous section, two LMMSE estimates of a PDS signal $x(n)$ can be obtained, respectively, in the horizontal and vertical directions, which are denoted by $\hat{x}_h(n)$ and $\hat{x}_v(n)$. Let $\hat{x}_h(n)$ and $\hat{x}_v(n)$ be the corresponding estimation errors, then

\[ \begin{cases} 
\hat{x}_h(n) = x(n) - \hat{x}_h(n) \\
\hat{x}_v(n) = x(n) - \hat{x}_v(n), 
\end{cases} \quad (4.1) \]

Either $\hat{x}_h(n)$ or $\hat{x}_v(n)$ exploits the correlation of $x(n)$ with its neighbors in a particular direction. A more accurate estimate of $x(n)$ can be obtained by fusing the two directional LMMSE estimates. We employ the weighted average strategy and let the fused estimate be

\[ \hat{x}_w(n) = \frac{\hat{x}_h(n) + w_v(n) \cdot \hat{x}_v(n)}{w_h(n) + w_v(n)} \quad (4.2) \]

where $w_h(n) + w_v(n) = 1$. The weights and $w_v(n)$ are determined to minimize the MSE of $\hat{x}_w(n)$

\[ \sigma^2_{x_w}(n) = E[(\hat{x}_w(n) - x(n))^2] = E[(x(n) - \hat{x}_w(n))^2] \quad (4.3) \]

or

\[ \sigma^2_{x_w}(n) = w_h^2(n) \cdot \sigma^2_{x_h}(n) + w_v^2(n) \cdot \sigma^2_{x_v}(n) \\
+ 2 \cdot w_h(n) \cdot w_v(n) \cdot E[\hat{x}_w(n) \cdot \hat{x}_w(n)] \quad (4.4) \]

Where $\sigma^2_{x_h}(n)$ and $\sigma^2_{x_v}(n)$ are the variances of estimation errors $\hat{x}_h(n)$ and $\hat{x}_v(n)$. Generally, the correlation between variables $\hat{x}_h$ and $\hat{x}_v$ is weak for a natural image, especially in the areas of edges and fine texture structures where the human visual system is sensitive to spatial resolution. In fact, if $\hat{x}_h$ and $\hat{x}_v$ are highly correlated, i.e., the two estimates $\hat{x}_h$ and $\hat{x}_v$ are close to each other, then varies little in $w_h$ and $w_v$ anyways.

Fig. 6.
(a) Blue sample and its four nearest red neighbors. (b) Red sample and its four nearest blue neighbors.

Assuming that $\hat{x}_h$ and $\hat{x}_v$ are approximately uncorrelated, the magnitude of the last term in the right side of (4.4) becomes negligible, or approximately

\[ \sigma^2_{x_w}(n) \approx w_h^2(n) \cdot \sigma^2_{x_h}(n) + w_v^2(n) \cdot \sigma^2_{x_v}(n) \\
= w_h^2(n) \cdot \left( \sigma^2_{x_h}(n) + \sigma^2_{x_v}(n) \right) \\
+ \sigma^2_{x_v}(n) - 2 \cdot w_h(n) \cdot \sigma^2_{x_v}(n). \quad (4.5) \]
To minimize $\sigma^2_{\hat{x}_w}(n)$, we set the partial differential of $\sigma^2_{\hat{x}_w}(n)$ with respect to $w_h(n)$ to zero

$$\frac{\partial \sigma^2_{\hat{x}_w}(n)}{\partial w_h(n)} = 2 \cdot w_h(n) \cdot (\sigma^2_{\hat{x}_w}(n) + \sigma^2_{\hat{x}_e}(n)) - 2 \cdot \sigma^2_{\hat{x}_w}(n) = 0$$

which yields

$$w_h(n) = \frac{\sigma^2_{\hat{x}_w}(n)}{\sigma^2_{\hat{x}_w}(n) + \sigma^2_{\hat{x}_e}(n)}, \quad w_e(n) = \frac{\sigma^2_{\hat{x}_e}(n)}{\sigma^2_{\hat{x}_w}(n) + \sigma^2_{\hat{x}_e}(n)}.$$  \hspace{1cm} (4.7)

Substituting (4.7) into (4.2) obtains $\hat{x}_w(n)$, the optimally weighted estimate of $\hat{x}_h(n)$ and $\hat{x}_e(n)$. The MSE of the optimal estimate $\hat{x}_w(n)$ is

$$\sigma^2_{\hat{x}_w}(n) = \frac{\sigma^2_{\hat{x}_w}(n)\sigma^2_{\hat{x}_e}(n)}{\sigma^2_{\hat{x}_w}(n) + \sigma^2_{\hat{x}_e}(n)},$$ \hspace{1cm} (4.8)

Obviously $\sigma^2_{\hat{x}_w}(n)$ is less than either of $\sigma^2_{\hat{x}_h}(n)$ and $\sigma^2_{\hat{x}_e}(n)$. Using the method described in Sections III and IV, we compute, for each red pixel position $R_n$, and each blue pixel position $B_n$, the directional weighted estimates of the green–red PDS $\Delta_{g,r}(n)$ and the green–blue PDS $\Delta_{g,b}(n)$. Then, we can recover the green channel of the Bayer CFA image by estimating the missing green samples as

$$\hat{C}_n = R_n + \Delta_{g,r}(n) \quad \text{or} \quad \hat{C}_n = B_n + \Delta_{g,b}(n).$$ \hspace{1cm} (4.9)

Compared with the red and blue channels of a Bayer CFA image, the green channel preserves much more detail of the image and, hence, is more important for the human visual system. Furthermore, the interpolation quality of red and blue channels, which is the subject of the next section, also depends on the estimation accuracy of the green channel.

V. Demosaicking Of The Chrominance Channels

In the previous two sections, we showed how to remove the demosaicking noise in the green channel by directional LMMSE.

**Fig. (7).** (a)–(b) Green sample and its two original and two estimated red neighbors. (c)–(d) Green sample and its two original and two estimated blue neighbors.
filtering of PDS and optimal fusing of the resulting directional LMMSE estimates. Once the robust green estimates are obtained for all pixels, they can guide, in conjunction with the PDS estimates, the
demosaicking of the red and blue channels. This is accomplished in the following two steps.

A. Interpolation of Missing Red (Blue) Samples at the Blue(Red) Sample Positions

We first interpolate the missing red sample at a blue pixel $B_p$. Referring to Fig. 6(a), we denote by $R_{n_w}$, $R_{n_s}$, $R_{n_e}$, and $R_{n}^w$ the four nearest red neighbors of the blue sample position $B_p$, where the superscripts are directional notations for northwestern, southwestern, northeastern, and southeastern. Note that $R_{n_w}$, $R_{n_s}$, $R_{n_e}$, and $B_p$ are all original samples in the Bayer pattern. The estimated green samples at these positions are denoted by $G_{n_w}$, $G_{n_s}$, $G_{n_e}$, and $G_{n}^w$, respectively. The available four green–red difference values are represented as $\Delta_{n_w,gr}$, $\Delta_{n_s,gr}$, $\Delta_{n_e,gr}$, and $\Delta_{n}^w,gr$. The estimate $\hat{R}_n$ of the missing red sample is to be computed. We interpolate the green–red PDS signal at the blue sample position $B_n$ as the average of the four available green–red differences, namely

$$\Delta_{n,gr} = \frac{\Delta_{n_w,gr} + \Delta_{n_s,gr} + \Delta_{n_e,gr} + \Delta_{n}^w,gr}{4}. \quad (5.1)$$

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TABLE III

PSNR (in Decibels) Results of the Proposed Method and the Other Methods
Fig. (8). Demosaicking results of image (1) in Fig. 2. (a) Original. (b) Method in [11]. (c) Method in [7]. (d) The proposed method.

Fig. (9). Demosaicking results of image (2) in Fig. 2. (a) Original. (b) Method in [11]. (c) Method in [7]. (d) The proposed method.

Then, the missing red sample is estimated as

\[ \hat{R}_n = \hat{G}_n - \Delta_{n,GR}, \quad (5.2) \]

Similarly, the missing blue samples at the red sample positions [referring to Fig. 6(b)] can be interpolated. The four green–blue difference values in the northwestern, southwestern, northeastern, and southeastern of are available, and they are averaged to interpolate the green–blue PDS signal at position. The missing blue sample is then estimated as

**B. Interpolation of Missing Red/Blue Samples at the Green Sample Positions**

After the missing red/blue samples at the blue/red positions have been filled, we arrive at the four cases depicted by Fig. 7. As before, the samples are estimated ones if marked by “\(^\ast\)”, and original ones otherwise. Due to the symmetry between red and blue samples in these four cases, we only need to discuss case (a). Given the green estimates \(G_n, G_s, G_e, \) and \(\hat{G}_n\) at the positions \(R_n, R_s, R_e, \) and \(\hat{R}_n\), we have the corresponding four green–red difference values, denoted by \(\Delta_{n,GR}, \Delta_{n,GR}, \Delta_{n,GR}, \) and \(\Delta_{n,GR}. \) As in the previous step, we compute the bilinear average of the green–red differences.
\[
\Delta_{m_{gr}} = \frac{(\Delta^R_{m_{gr}} + \Delta^G_{m_{gr}} + \Delta^B_{m_{gr}} + \Delta^W_{m_{gr}})}{4}.
\]  

(5.3)

Then, the missing red sample at green sample position \(G_n\) is estimated to be \(R_n = G_n - \Delta_{m_{gr}}\). Similarly, the missing blue sample at a green position \(G_n\) is estimated as \(B_n = G_n - \Delta_{m_{gr}}\). By now, we have filled in all the missing red and blue samples. The full color image is reconstructed. The presented demosaicking scheme first exploits the correlation between the green and red/blue channels to obtain good estimates of the missing green samples and then estimates the missing red and
VI. Experimental Results

We implemented the proposed LMMSE color demosaicking algorithm, and tested it on a large number of natural color images. In this section, we present our experimental results for the eighteen images in Fig. 2, and compare them with the methods of Hamilton et al. [11], Chang et al. [7], and Gunturk et al. [9], which are among the best schemes. The results reported in the recent paper of [9] were better than the previously published algorithms, especially for the red and blue channels. In the implementation of our scheme, the standard deviation of the Gaussian smooth filter, [referring to (3.5)], was set around 2, and the parameter [referring to (3.6) and (3.9)] was set to 4. In Table III, the peak signal to noise ratios (PSNR) of the demosaicked images by the four methods are listed. The results of the method in [9] are duplicated from that paper. They were originally reported by MSE, and we transformed them into PSNR by $PSNR = 10 \log_{10}(255^2/MSE)$.

It can be seen from Table III that the estimates of the green channel are significantly improved by the proposed demosaicking algorithm. On average the improvement is 4.74, 4.04, and 2.24 dB over the other three algorithms, respectively, in PSNR. The new algorithm also outperforms the other algorithms in red and blue channels as well. The margins of improvement in PSNR are 5.87 and 6.23 dB over the algorithm of [11] and the algorithm of [7] for the red channel, and, respectively, 5.06 and 5.46 dB for the blue channel. Compared with the algorithm of [9], the new algorithm achieves 0.46-dB higher PSNR in the red channel and 0.84-dB higher PSNR in the blue channel. One should keep in mind that the demosaicking results of [9] in the red and blue channels were obtained by costly eight iterations of wavelet-based filtering operations, while our results were obtained by simple bilinear interpolation of the primary difference signals. The computation and implementation complexities are considerably lower than [9]. In Figs. 8–11, some samples of the original and the demosaicked images by different methods ([11], [7], and the proposed) are shown for the purpose of subjective quality evaluation. For the visual results of [9], the reader can refer to the original paper. The proposed LMMSE-based demosaicking algorithm appears to produce visually more pleasant color images with color artifacts greatly suppressed.

VII. Conclusion

This paper presented a new color demosaicking technique of LMMSE directional filtering of the green–red and green–blue PDS signals. The missing green samples are estimated from the filtered PDS in both horizontal and vertical directions, and the two estimates are optimally fused. The resulting green channel is then used to guide the estimation of the missing red and blue samples.

VIII. References


Biographies

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